Estimating Mortality Rates: The Role of Proportional Life Expectancy

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The computation of long-term survival is usually based on adjustments to the conventional life table. Assessing the validity of different types of adjustments can be difficult, partly because of the need to allow for two age-related trends—the decline in the average (normal) life expectancy, as well as in the new (abnormal) estimate. In this paper, we illustrate the value of routinely expressing each new estimate as a percentage of normal at each age. An additional finding has been that in some common disorders this proportional life expectancy (PLE) remains remarkably constant over many years.

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Estimates of individual life expectancy are needed to help address problems arising in clinical, legal and insurance contexts. Several methods of producing these estimates have been proposed. Regardless of the method used, it can be useful to examine results by calculating the proportional life expectancy (PLE), the estimated life expectancy (in years) as a percentage of the normal life-table value for the same age.

This simple calculation is especially useful when there is a large age difference. For example, in a certain disorder the life expectancy estimate at age 50 may be only half that of a patient aged 20. However, the decline in normal life expectancy over this age range is also likely to be close to 50%. This means that, relative to normal life expectancy, there may be little or no change in proportional life expectancy.

Recent figures from a number of large studies have shown that in some conditions the PLE remains at or close to a constant figure over an age range of at least 40 or 50 years. In others there may be a constant linear trend.

The long-term effects of spinal cord injury have been the subjects of much research in the last 30 years, and several publications have included detailed tables of life expectancy. This provides a convenient framework within which to examine the changes that have taken place in the way that long-term survival can best be computed.

STUDIES USING A SINGLE MORTALITY RATIO

Smart and Sanders 1976

In 1976, the Insurance Institute for Highway Safety published The Costs of Motor Ve-
Figure 1. Trends in predicted life expectancy in persons with spinal cord injury. Expressed as a percentage of the normal value at each age (the “proportional life expectancy”). Legend: Inc. para: incomplete paraplegia (Mortality Ratio 1.18), Inc. quad: incomplete quadriplegia (Mortality Ratio 2.16), Comp. para: complete paraplegia (Mortality Ratio 4.00), Comp. quad: complete quadriplegia (Mortality Ratio 12.00). Data from Smart and Sanders, 1976.1

Vehicle Related SCI by Charles Smart and Claudia Sanders.1 This is a useful starting point for the present paper, since most of the other articles that will be quoted have referred to this publication under the heading of Method.

The life expectancy calculations in this article are based on the single (all-age) mortality ratios (MR) reported from a follow-up study in Toronto, Ontario.2 Persons with spinal cord injury (SCI) had been grouped into four categories—quadriplegia and paraplegia, each subdivided into complete and incomplete lesions. Mortality ratios ranged from a low of 1.18 to a high of 12.00, and downward trends are seen in all categories when the PLE figures are plotted against age (Figure 1).

The group with partial paraplegia (MR 1.18) showed the least change in proportional life expectancy, with a PLE of 95% at age 20, declining to 80% by age 70. The most severely affected were those with complete quadriplegia (MR 12.00) with a PLE of 50% at age 20, declining to 20% by age 70. The other two categories had curves that lay between these two extremes. Elsewhere in this paper, the more severe categories will be emphasized, partly because they have the largest financial implications, and also because they show the most clear-cut changes in PLE with age.

In passing, it should be noted that the level and slope of the lines in Figure 1 are not unique to spinal cord injury but would be the same for any single (all-age) MR of 1.18, 12.00, etc. regardless of the underlying disorder.

Geisler 1983

In 1983 the Toronto group published its fourth SCI report, and for the first time the calculations excluded patients who died within the first year after injury.3 This was done to avoid underestimating long-term survival in those with severe injuries, in whom there was a high death rate in the first year. Largely as a result of this change, the MR for complete quadriplegia fell from the previously indicated 12.00 to 7.67.

Due to the growing recognition of the usefulness of excess death rates (EDR) in mortality studies,4,5 this study also provided the EDR for each category. (An EDR provides the difference between the observed and expected death rates, rather than their ratio, as seen in the MR). Adding an EDR to the death rates in the life table has an effect on PLE that is the opposite of that caused by the MR, with the PLE (EDR) estimates rising at almost the same upward slope as the PLE (MR) estimates decline (Figure 2).

The underlying reason for this pattern is that normal death rates rise very rapidly after age 40, and multiplication of these rates by a fixed number has a large effect on life expectancy. In contrast, the addition of a single fixed rate becomes of less and less relative importance, as the underlying normal rate gets larger, so that the PLE (EDR) figure rises steadily toward normal (100%) with increasing age.6
Figure 2. Trends in proportional life expectancy in persons with complete quadriplegia, using a mortality ratio (MR) of 7.67, or an excess death rate (EDR) of 0.0244. Data from Geisler et al, 1983.3

Figure 3. Trend in proportional life expectancy, using mortality ratios based on either 4 age-groups, or 1 age-group (all ages). Based on data for C1-4 (Frankel ABC). Data from DeVivo et al, 1995.7

STUDIES USING MORE THAN ONE MORTALITY RATIO

DeVivo 1995

In 1995, DeVivo et al published survival data on over 17,000 subjects from the US Model Systems study of SCI.7 This publication contained detailed estimates of life expectancy at 5-year intervals from ages 5 to 80. In addition, the data were broken down into 4 age groups: under 31, 31–45, 46–60, and over 60.

There were still 4 diagnostic categories, but they were now redefined. The first two were for high and low tetraplegia (C1-4 and C5-8, and previously called quadriplegia). The third category was paraplegia (T1-S5). The fourth was Frankel D (incomplete, motor functional) and included lesions at all levels (C1-S5). The first 3 groups now included cases that were Frankel A (complete lesions, as before); plus Frankel B and C (sensory sparing only, and motor non-functional).

Figure 3 shows the effect on PLE in the C1-4 group that results from applying the four MR to the life table, compared to the use of a single (all-age) MR calculated from the same source. As usual, the PLE from a single all-age MR show a steady downward trend, in this case from 65% of normal at age 20 to 25% by age 80.

In contrast, the line based on the four MR shows almost no decline in PLE between age 20 and 60 (58% down to 54%), followed by a decline that is parallel to the line based on the single all-age MR. This pattern reflects the fact that with MR based on relatively small age groups the PLE changes little between age 20 and age 60, but after age 60 there is (again) only a single MR at all ages and a steep decline from age 60 (54%) to age 80 (38%).

Anderson 2002

A similar pattern has been reported in 4 unrelated conditions for which both MR and EDR were available by 10-year age groups to age 60 or 70. Figure 4a shows the PLE estimates based on both MR and EDR data for one of these conditions (males more than 30% overweight).6(p69)

It should be noted that not only are the PLE values almost constant from age 40 to 70, but the data points are virtually identical for both the MR and EDR calculations. This is very different from the divergent values seen in Figure 2. This reflects the fact that even with 10-year age groups the MR and EDR have
Figure 4a. Proportional life expectancy in overweight persons, using either excess death rates (EDR), or mortality ratios (MR). The MR and EDR were based on death rates in 10-year age-groups below age 70, then a single age-group from age 70 to 109. From Anderson, 2002. Figure reprinted with permission from Tewit Press.

Figure 4b. Trends in proportional life expectancy in four conditions: overweight (same data as in Figure 4a); cerebral palsy; diabetes; and alcoholism. From Anderson, 2002.

Figure 5. Trends in proportional life expectancy in persons with 4 types of spinal cord injury (see text for full description). Regression lines have been based on the least squares method. Legend: Frankel D: (C1-S5) incomplete lesion, motor functional, Paraplegia: (T1-S5) Frankel ABC, C5-8: low tetraplegia Frankel ABC, C1-4. high tetraplegia Frankel ABC, Reg’n: regression line. Data from Strauss et al, 2000.

almost reached the point at which their effects would be identical (ie, where data are available for each single year of age).

A summary graph of all 4 conditions is shown in Figure 4b, with regression lines based on the method of least squares. Little or no gradient is seen in the top 3 conditions, but there is a distinct upward trend in the lowest PLE line (for alcoholism) at around 3% per decade.

Strauss 2000

In 2000, Strauss et al published a further analysis of the Model Systems data, with MR calculated for 14 five-year age groups (10±14 to 75±79), plus the final open-ended group of 80 plus. There were few cases or deaths at the extremes of age. However, between ages 20 and 70, there were at least 20 deaths in most age groups, enough to give reasonably stable MR.

The life expectancy estimates between 20 and 70 are plotted in Figure 5 as PLE, again with regression lines based on least squares. Three of the regression lines are remarkably flat, with very little change over the 50 years between ages 20 and 70. However, in contrast to Figure 4b, the exception is not in the most severe category (C1-4) but in the intermediate category of C5-8. Also, although the slope is downward rather than upward, the gradient
Figure 6. Estimates of proportional life expectancy values from age 30 to 90. The top line is based on the declining log method of Strauss, in which a hypothetical MR of 4.7 at age 30 has been used to create a proportional life expectancy of 77%. For comparison, the lower line shows a constant PLE of 75%.

is around the same at approximately 3% per decade.

OTHER METHODS

Strauss 1998

Strauss and colleagues have suggested that the distortion produced by using a fixed MR at each year of age can be avoided by gradually reducing the MR once the steep rise in the normal (average) death rates of the life table appears after age 30. Taking into account the exponential nature of this rise (doubling about every 8 years), the MR is first converted to its logarithm, then reduced steadily until the rate becomes normal (ie, when the MR becomes 1.0). This is usually assumed to occur between age 85 and 100.

An example is shown in Figure 6, beginning with a hypothetical MR of 4.7 at age 30, reaching parity at age 90. The initial PLE is approximately 77% at age 30, and rises to reach a plateau of 100% at age 90.

For comparison, a horizontal line has been added to Figure 6, just under the 77% level, to illustrate an interesting feature. The PLE based on the "declining MR" estimate of life expectancy is constant for the first 30 years, and then begins to rise toward its target of 100%. (See Discussion).

Two different statistical models have been used in recent SCI articles in which data smoothing was required to help clarify the results. Unfortunately, in each case this has led to some distorted figures in the elderly. Although the distortions are not easily seen in the tabular data, the problems become obvious when graphs are plotted of PLE values against age. We have also found that these issues are of practical as well as theoretical importance, despite the fact that fewer than 5% of SCI patients are over 60 when injured.

 DeVivo 1995

In an elegant study of ventilator-dependent patients, DeVivo and Ivie provided a table of life expectancy values that included estimates for persons who had survived for at least 1 or 2 years after injury. In each case the calculations were first carried out using MR from 3 age-groups: below 30; 30–60; and over 60. The resulting PLE values are shown in Figure 7a by large symbols and broken lines, and it can be seen that the trends for the 1-year and 2-year survivors are reasonably similar.

Linear regression of age against the square root of life expectancy had then been used to smooth the life expectancy values within each group. In the 1-year survivors, the PLE estimates follow an almost flat line at around 35% (the lowest line in the Figure). However, in the 2-year survivors, the same smoothing process creates a PLE trend that rises to 80% of normal by age 80 and if extended would reach 100% (ie, normal) by age 90.

Frankel 1998

A second example of the potential risks of statistical smoothing comes from an outstanding 50-year English study, in which polynomial curves were used to overcome the instability of the four age-specific MR.
These MR had been calculated for each of three SCI categories in an analysis that was comparable to the large 1995 study by DeVivo et al. (A single category of C1-8 was used for tetraplegia.)

The authors were handicapped by relatively small numbers compared to the DeVivo study, which had involved approximately 10 times as many subjects over the same time period (1973 to 1990). In addition, this was the first study to provide detailed life expectancy estimates separately for males and females. This further reduced the size of the available groups, especially in females, who only account for about 20% of all SCI patients.

The polynomial curves that were used were successful in smoothing the life expectancy estimates up to about age 60, but after this the slope (and even direction) of each MR trend was a matter of chance, determined largely by the relative size of the third and fourth MR values in each category.

The distortion was particularly severe for persons with paraplegia, since the polynomial curve took the MR steeply upward beyond age 70, thus leading to a correspondingly rapid fall in PLE. As can be seen in Figure 7b, the male and female PLE were down to 20% by age 80 and should reach zero (ie, no survival) by about age 83.

**DISCUSSION**

It is barely 10 years since the first detailed tables of life expectancy in patients with SCI were published in 1995, but they have contributed to an increased understanding of the issues affecting the prediction of long-term survival in persons with chronic, life-threatening disease.

It is now clear that from the point of view of calculating life expectancy, one of the main advantages of any large study is the ability to provide several age-specific MR (or EDR) that, based on the age interval chosen, include a sufficiently large number of deaths to ensure a narrow confidence interval (say, at most, 10–15 years in width). This will usually...
still leave a large final age group above age 70 or 80, but the assumption of a fixed PLE (or a fixed trend in PLE) should be reasonably accurate in most cases.

The finding that the declining MR method of Strauss et al generates a constant PLE for the first few decades suggests that both methods may be approaching a common, workable solution to the problem of estimating life expectancy in elderly patients. At this stage, we think a temporary solution might be to generate an initial life expectancy estimate using the Strauss method, and then convert the life expectancy in the first year to a PLE. This constant PLE could then be applied to the rest of the life table, using a formula that Strauss et al have kindly suggested to us and that appears in another article in this issue of the Journal.¹³

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REFERENCES